

Polaron formation in the vicinity of a narrow Feshbach resonance in atomic and exciton-polariton condensates.

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Abstract

The polaronic system consisting of an impurity in a dilute Bose-Einstein condensate is considered in the presence of a narrow Feshbach resonance. For this purpose a coupled-channel model is used, which at the mean field level predicts the formation of quasiparticles that are a superposition of the impurity and the molecular states. The impurity-boson interactions and the coupling between the open and closed channels are then considered weak and a perturbative calculation of the corrections to the mean field results is presented. This allows to examine the properties of the quasiparticles, such as the lifetime and the effective mass. The model is applied to two physical systems: an impurity atom in a Bose-condensed atomic gas in 3D and a spin down lower polariton in a Bose-Einstein condensate of spin up lower polaritons in 2D. The model parameters are linked to the physical parameters by identifying the low energy T -matrix and applying a proper renormalization scheme.

I. INTRODUCTION

In recent years ultracold atomic systems have become increasingly popular as quantum simulators for many-particle models [1]. These systems are attractive since they are extremely clean as compared to a typical solid state experiment and the extreme controllability and tunability. One can, for example, experimentally vary the interatomic interactions by means of a Feshbach resonance. This is a scattering resonance which occurs when the scattering energy approaches the binding energy of a molecular state (see for example Ref. [2] for a review on Feshbach resonances in ultracold gases). If the molecule and the scattering atoms have a different magnetic moment, the difference in energy can be varied with an external magnetic field. This allows to experimentally tune the interatomic interactions as a function of an external magnetic field. A distinction is typically made between a broad or entrance channel dominated resonance and a narrow or closed channel dominated resonance [2]. A broad resonance can be well-described by a single channel model with an effective interaction amplitude while for the description of a narrow resonance the molecular state has to be described explicitly with a coupled-channel model.

Recently the principle of quantum simulation with ultracold gases has been considered for the Fröhlich polaron [3–5]. The polaron is well-known from solid state physics for the description of a charge carrier in a charged lattice, described by the Fröhlich Hamiltonian. If the Bogoliubov approximation is valid the Hamiltonian of an impurity in a Bose-Einstein condensation can be mapped onto the Fröhlich Hamiltonian [3–5]. This set-up is particularly attractive since the polaronic coupling parameter depends on the interatomic interaction strength which is tunable with a Feshbach resonance, whereas in the solid state context this is a material constant. An example of a promising experiment to probe the polaronic strong coupling regime is the doping of an ultracold Rb gas with single Cs impurity atoms from Ref. [6] since it exhibits a broad interspecies Feshbach resonances [7]. On the other hand there are also binary systems that exhibit only narrow interspecies Feshbach resonances, as for example a ^6Li -Na mixture [8], in which case the influence of the resonance on the polaronic properties is not clear.

A related research field that has attracted a great deal of interest in the context of quantum simulation are quantum fluids of light (See for example Ref. [9] for a review). If a semiconductor is placed in a cavity with a strong coupling between the cavity modes and

the excitons they combine to form new quasiparticles, known as polaritons. The coupling strength is given by the Rabi frequency. By using a cavity with a position dependent thickness it is possible to experimentally tune the exciton-photon detuning by probing a different area of the cavity. Since the effective mass of the polaritons is several orders of magnitude smaller than the exciton mass, they form a Bose-Einstein condensate at much higher temperatures. The cavity inevitably exhibits losses, which means that to accomplish a steady state the system has to be constantly pumped and is never at equilibrium. However, if the loss rate is sufficiently slow as compared to the thermalization time of the polariton gas a quasithermal equilibrium can be achieved. By introducing two different circularly polarized cavity modes which couple to excitons with different spins it is possible to create spinor polaritons with two spin states. By using different light intensities for the polarizations a population imbalance can be introduced between the two spin states which in the extreme case results in a single spin down lower polariton in a sea of spin up lower polaritons. If the spin up lower polaritons form a dilute Bose-Einstein condensate the resulting Hamiltonian can, in the same manner as for the ultracold gas system, be mapped onto the Fröhlich polaron Hamiltonian. However with the distinction that due to the cavity losses the system is not at equilibrium. Since two excitons can combine to form a biexciton also these systems exhibit Feshbach resonances which have been theoretically predicted in Refs. [10, 11] and confirmed experimentally in Ref. [12].

In this paper we are particularly interested in the polaronic system in the presence of a narrow Feshbach resonance in the two physical contexts outlined above. Both systems are considered at equilibrium which is expected to be a good description for an ultracold atomic gas but may be a rather crude approximation for current experiments on the polaritonic system. A narrow Feshbach resonance corresponds to weak coupling to the molecular channel so that perturbation theory is applicable to describe the corrections to the mean field results. The experimental relevance of this regime is demonstrated by the wide range of resonance widths that have been observed with ultracold gases, including narrow resonances [2] and the recent observations of a narrow polaritonic Feshbach resonance [12].

The structure of this manuscript is as follows. First the model Hamiltonian describing two types (denoted as spin up and down) of particles that can form a molecule is introduced. The manifestation of a Feshbach resonance within this model is investigated by calculating the two-body T -matrix and a renormalization scheme is discussed. Then, the

spin up particles are assumed to form a dilute Bose-Einstein condensate by applying the Bogoliubov approximation. The mean field part of the Hamiltonian is then diagonalized with another Bogoliubov transformation resulting in two types of quasiparticles consisting of a superposition of the spin down particles and the molecules. The interactions are then assumed to be weak and perturbation theory is used to calculate the lowest order corrections to the self energies of the quasiparticles. The model is then applied to two physical systems: an impurity atom in a Bose-condensed atomic gas in 3D and a spin down lower polariton in a Bose-Einstein condensation of spin up lower polaritons in 2D. The results are numerically studied for these two systems and finally the conclusions are presented.

II. THE MODEL SYSTEM

The following Hamiltonian is considered for the model system:

$$\begin{aligned}
\hat{H} = & \sum_{\vec{k}} \sum_{\sigma=\uparrow,\downarrow} \xi_{\vec{k},\sigma} \hat{a}_{\vec{k},\sigma}^\dagger \hat{a}_{\vec{k},\sigma} + \sum_{\vec{k}} \varepsilon_{\vec{k}}^M \hat{b}_{\vec{k}}^\dagger \hat{b}_{\vec{k}} \\
& + g \sum_{\vec{k},\vec{q}} \left(\hat{b}_{\vec{k}}^\dagger \hat{a}_{\frac{\vec{k}}{2}+\vec{q},\uparrow} \hat{a}_{\frac{\vec{k}}{2}-\vec{q},\downarrow} + h.c. \right) + \frac{U^{\uparrow\uparrow}}{2} \sum_{\vec{k},\vec{k}',\vec{q}} \hat{a}_{\vec{k}+\vec{q},\uparrow}^\dagger \hat{a}_{\vec{k}'-\vec{q},\uparrow}^\dagger \hat{a}_{\vec{k}',\uparrow} \hat{a}_{\vec{k},\uparrow} \\
& + U^{\uparrow\downarrow} \sum_{\vec{k},\vec{k}',\vec{q}} \hat{a}_{\vec{k}+\vec{q},\uparrow}^\dagger \hat{a}_{\vec{k}'-\vec{q},\downarrow}^\dagger \hat{a}_{\vec{k}',\downarrow} \hat{a}_{\vec{k},\uparrow}.
\end{aligned} \tag{1}$$

The first term represents the kinetic energy of the spin up ($\sigma = \uparrow$) and spin down ($\sigma = \downarrow$) particles with $\hat{a}_{\vec{k},\sigma}^\dagger$ ($\hat{a}_{\vec{k},\sigma}$) the creation (annihilation) operators for a particle with wave vector \vec{k} , spin σ and $\xi_{\vec{k},\sigma} = E_{\vec{k},\sigma} - \mu_\sigma$, where μ_σ is the chemical potential and $E_{\vec{k},\sigma} = E_{0,\sigma} + \hbar^2 k^2 / (2m_\sigma)$ the energy with m_σ the mass. The second term gives the energy of the free molecules, consisting of a bound state of a spin up and a spin down particle, with $\hat{b}_{\vec{k}}^\dagger$ ($\hat{b}_{\vec{k}}$) the creation (annihilation) operators for a molecule with wave vector \vec{k} and $\varepsilon_{\vec{k}}^M = -\varepsilon^M + \hbar^2 k^2 / (2M) - \mu_\uparrow - \mu_\downarrow$, with ε^M the binding energy and M the mass. The first term on the second line represents the formation and dissociation of the molecule with interaction amplitude g . The next term is the interaction energy of the spin up particles interacting with a contact potential with amplitude $U^{\uparrow\uparrow}$ and the last term represents the interaction between the spin up and spin down particles for which also a contact potential is considered, with amplitude $U^{\uparrow\downarrow}$. Note that no interactions between the spin-down particles are considered.

A. Two body T -matrix and renormalization

Only considering one spin up and one spin down particle allows a calculation of the two-body T -matrix of the model system with a derivation along the lines of Ref. [13]. This results in the following expression:

$$\frac{1}{T^{\uparrow\downarrow}(E)} = \left(U^{\uparrow\downarrow} + \frac{|g|^2}{E - \varepsilon^M} \right)^{-1} - \sum_{\vec{q}} \frac{1}{E - \frac{\hbar^2 q^2}{2m_r} + i\delta}; \quad (2)$$

with E the scattering energy, δ a positive infinitesimal and m_r the reduced mass: $m_r^{-1} = m_{\downarrow}^{-1} + m_{\uparrow}^{-1}$. Note that if the dimension is larger than 1 this expression contains an UV divergence which can be renormalized with the scheme introduced in Ref. [13] for a 3 dimensional set-up and later also applied in Ref. [14] for a quasi-two-dimensional atomic gas. This results in the following renormalization scheme to express the two-body model parameters $U^{\uparrow\downarrow}$, g and ε^M in terms of the corresponding physical parameters $U_P^{\uparrow\downarrow}$, g_P and ε_P^M :

$$U^{\uparrow\downarrow} = \frac{U_P^{\uparrow\downarrow}}{1 + RU_P^{\uparrow\downarrow}}; \quad (3a)$$

$$g = \frac{g_P}{1 + RU_P^{\uparrow\downarrow}}; \quad (3b)$$

$$\varepsilon^M = \varepsilon_P^M + Rgg_P; \quad (3c)$$

with:

$$R = \sum_{\vec{q}} \frac{1}{E - \frac{\hbar^2 q^2}{2m_r}}. \quad (4)$$

Expressing the T -matrix (2) as a function of the physical parameters results in an expression that does not contain an UV divergence:

$$T^{\uparrow\downarrow}(E)^{-1} = \left(U_P^{\uparrow\downarrow} + \frac{|g_P|^2}{E - \varepsilon_P^M} \right)^{-1} + i\pi g(E); \quad (5)$$

where $g(E)$ is the density of states for a free gas with mass m_r . Note that the $|g_P|^2 / (E - \varepsilon_P^M)$ term in (5) describes the Feshbach resonance.

B. Coupled-channel versus single-channel model

As mentioned in the introduction a distinction is typically made between broad or entrance channel dominated and narrow or closed channel dominated resonances for Feshbach

resonances [2]. As the name suggests the molecular channel is important for the closed channel dominated resonances and they should be described with a coupled-channel model, as for example described by Hamiltonian (1). For a narrow or entrance channel dominated resonance the occupation of the closed molecular channel is typically small and a single-channel model can be used with an effective interaction amplitude $U_{eff}^{\uparrow\downarrow}$ for the description:

$$U_{eff}^{\uparrow\downarrow} = U_P^{\uparrow\downarrow} - \frac{|g_P|^2}{\varepsilon_P^M}. \quad (6)$$

In the following a coupled-channel calculation is developed and analyzed in perturbation theory in g_P . Its results will be compared to a perturbative single-channel calculation.

III. BOGOLIUBOV APPROXIMATION

The temperature is assumed to be sufficiently low such that a macroscopic number of spin up particles occupy the one-particle ground state and thus form a Bose Einstein condensation. If the gas is dilute, the Bogoliubov approximation can be used which transforms the Hamiltonian (1) into [15]

$$\hat{H} = E^{GP} + \hat{H}^{MF} + \hat{H}_I. \quad (7)$$

The first term is the well-known Gross-Pitaevskii energy of the condensate [15]. The second term is the mean field Hamiltonian, which consists of all quadratic contributions:

$$\hat{H}^{MF} = \sum_{\vec{k}} \begin{pmatrix} \hat{a}_{\vec{k}}^\dagger & \hat{b}_{\vec{k}}^\dagger \end{pmatrix} \begin{pmatrix} \xi_{\vec{k},\downarrow} + U^{\uparrow\downarrow} n_0 & g\sqrt{n_0} \\ g\sqrt{n_0} & \varepsilon_{\vec{k}}^M \end{pmatrix} \begin{pmatrix} \hat{a}_{\vec{k}} \\ \hat{b}_{\vec{k}} \end{pmatrix} + \sum_{\vec{k} \neq 0} \varepsilon_{\vec{k}}^{Bog} \hat{\alpha}_{\vec{k}}^\dagger \hat{\alpha}_{\vec{k}}. \quad (8)$$

The condensate density is denoted as n_0 and the operators $\{\hat{\alpha}_{\vec{k}}^\dagger\}$ and $\{\hat{\alpha}_{\vec{k}}\}$ create and annihilate the Bogoliubov excitations with the corresponding Bogoliubov dispersion:

$$\varepsilon_{\vec{k}}^{Bog} = \sqrt{\left(E_{\vec{k}}^\uparrow + U^{\uparrow\uparrow} n_0\right)^2 - (U^{\uparrow\uparrow} n_0)^2}. \quad (9)$$

Where the notation $E_{\vec{k}}^\sigma = E_{\vec{k},\sigma} - E_{0,\sigma}$ was introduced. The operators $\hat{\alpha}_{\vec{k}}^\dagger$ and $\hat{\alpha}_{\vec{k}}$ are related to the spin up operators with the Bogoliubov transformation (for $\vec{k} \neq \vec{0}$):

$$\begin{aligned} \hat{\alpha}_{\vec{k}}^\dagger &= u_k \hat{a}_{\vec{k},\uparrow}^\dagger + v_k \hat{a}_{-\vec{k},\uparrow}; \\ \hat{\alpha}_{\vec{k}} &= u_k \hat{a}_{\vec{k},\uparrow} + v_k \hat{a}_{-\vec{k},\uparrow}^\dagger. \end{aligned} \quad (10)$$

The coefficients u_k and v_k are given by:

$$v_k^2 = \frac{1}{2} \left(\frac{E_k^\uparrow + U^{\uparrow\uparrow} n_0}{\varepsilon_{\vec{k}}} - 1 \right); \quad (11)$$

$$u_k^2 = \frac{1}{2} \left(\frac{E_k^\uparrow + U^{\uparrow\uparrow} n_0}{\varepsilon_{\vec{k}}} + 1 \right). \quad (12)$$

For the chemical potential of the condensate the lowest order approximation $\mu_\uparrow = E_{0,\uparrow} + U^{\uparrow\uparrow} n_0$ is used. Note that this induces a shift of the molecular energy $\varepsilon_{\vec{k}}^M$. A typical length scale of the condensate is the healing length, defined as: $\xi = \hbar (mn_0 U^{\uparrow\uparrow})^{-1/2}$.

The interaction Hamiltonian \hat{H}_I is given by

$$\hat{H}_I = \sum_{\vec{k}, \vec{q} \neq 0} \left[\left(g \hat{b}_{\vec{q}+\vec{k}}^\dagger \hat{a}_{\vec{k}} + U^{\uparrow\downarrow} \sqrt{n_0} \hat{a}_{\vec{k}+\vec{q}}^\dagger \hat{a}_{\vec{k}} \right) \left(u_q \hat{\alpha}_{\vec{q}} - v_q \hat{\alpha}_{-\vec{q}}^\dagger \right) + h.c. \right]. \quad (13)$$

From now on only a single spin down particle is considered which gives for the corresponding chemical potential $\mu_\downarrow = E_{0,\downarrow}$.

IV. DIAGONALIZATION OF THE MEAN FIELD PART

The mean field Hamiltonian (8) can be diagonalized by applying the following unitary transformation:

$$\begin{pmatrix} \hat{a}_{\vec{k}}^\dagger \\ \hat{b}_{\vec{k}}^\dagger \end{pmatrix} = \begin{pmatrix} \beta_{\vec{k}} & -\alpha_{\vec{k}} \\ \alpha_{\vec{k}} & \beta_{\vec{k}} \end{pmatrix} \begin{pmatrix} \hat{\Phi}_{\vec{k}}^\dagger \\ \hat{\Psi}_{\vec{k}}^\dagger \end{pmatrix}. \quad (14)$$

with:

$$\beta_{\vec{k}}^2 = \frac{1}{2} \left(1 + \frac{E_{\vec{k}}^\downarrow + U^{\uparrow\downarrow} n_\uparrow - \varepsilon_{\vec{k}}^M}{\sqrt{\left(E_{\vec{k}}^\downarrow + U^{\uparrow\downarrow} n_0 - \varepsilon_{\vec{k}}^M \right)^2 + 4g^2 n_0}} \right); \quad (15a)$$

$$\alpha_{\vec{k}}^2 = \frac{1}{2} \left(1 - \frac{E_{\vec{k}}^\downarrow + U^{\uparrow\downarrow} n_\uparrow - \varepsilon_{\vec{k}}^M}{\sqrt{\left(E_{\vec{k}}^\downarrow + U^{\uparrow\downarrow} n_0 - \varepsilon_{\vec{k}}^M \right)^2 + 4g^2 n_0}} \right). \quad (15b)$$

The mean field Hamiltonian (8) is then transformed into:

$$\hat{H}^{MF} = \sum_{\vec{k}} \left(\varepsilon_{\vec{k}}^+ \hat{\Phi}_{\vec{k}}^\dagger \hat{\Phi}_{\vec{k}} + \varepsilon_{\vec{k}}^- \hat{\Psi}_{\vec{k}}^\dagger \hat{\Psi}_{\vec{k}} + \varepsilon_{\vec{k}}^{Bog} \hat{\alpha}_{\vec{k}}^\dagger \hat{\alpha}_{\vec{k}} \right). \quad (16)$$

This shows the emergence of two quasiparticles that consist of a superposition the impurity and the molecular state, with dispersions:

$$\varepsilon_{\vec{k}}^{\pm} = \frac{1}{2} \left(E_{\vec{k}}^{\downarrow} + U^{\uparrow\downarrow} n_0 + \varepsilon_{\vec{k}}^M \pm \sqrt{\left(E_{\vec{k}}^{\downarrow} + U^{\uparrow\downarrow} n_0 - \varepsilon_{\vec{k}}^M \right)^2 + 4g^2 n_0} \right). \quad (17)$$

The mean field effective masses $m^{MF(\pm)}$ can be determined from the behavior of the dispersion at small $k = |\vec{k}|$:

$$\frac{1}{m^{MF(\pm)}} = \frac{1}{\hbar^2} \left. \frac{\partial^2 \varepsilon_{\vec{k}}^{\pm}}{\partial k^2} \right|_{k \rightarrow 0}. \quad (18)$$

Applying the unitary transformation (14) for the interaction Hamiltonian (13) results in:

$$\hat{H}_I = \sum_{\vec{q} \neq 0, \vec{k}} \left(\gamma_{\vec{k}+\vec{q}} \hat{\Phi}_{\vec{k}+\vec{q}}^{\dagger} + \sigma_{\vec{k}+\vec{q}} \hat{\Psi}_{\vec{k}+\vec{q}}^{\dagger} \right) \left(\beta_{\vec{k}} \hat{\Phi}_{\vec{k}} - \alpha_{\vec{k}} \hat{\Psi}_{\vec{k}} \right) \left(u_q \hat{\alpha}_{\vec{q}} - v_q \hat{\alpha}_{-\vec{q}}^{\dagger} \right) + h.c.; \quad (19)$$

where the following functions were introduced:

$$\gamma_{\vec{k}} = g\alpha_{\vec{k}} + U^{\uparrow\downarrow} \sqrt{n_0} \beta_{\vec{k}}; \quad (20)$$

$$\sigma_{\vec{k}} = g\beta_{\vec{k}} - U^{\uparrow\downarrow} \sqrt{n_0} \alpha_{\vec{k}}. \quad (21)$$

V. PERTURBATIVE CALCULATION OF THE SELF ENERGY

In general the Green's function of a particle with creation (annihilation) operators $\{\hat{\Psi}_{\vec{k}}^{\dagger}\}$ ($\{\hat{\Psi}_{\vec{k}}\}$) is defined as [16]:

$$G(\vec{k}, t - t') = -i \langle \mathcal{T} \hat{\Psi}_{\vec{k}}(t) \hat{\Psi}_{\vec{k}}^{\dagger}(t') \rangle. \quad (22)$$

Where \mathcal{T} is the time-ordering operator. The frequency representation of the Green's function is given by the Fourier transform with respect to the time:

$$G(\vec{k}, \omega) = \int_{-\infty}^{\infty} dt e^{i\omega t} G(\vec{k}, t) \quad (23)$$

and can typically be written as a function of the self energy $\Sigma(\vec{k}, \omega)$:

$$G(\vec{k}, \omega) = \frac{\hbar}{\hbar\omega - \varepsilon_{\vec{k}} - \hbar\Sigma(\vec{k}, \omega)}. \quad (24)$$

In appendix A some details are provided on the perturbative calculation of the lowest non-vanishing contribution of the interaction Hamiltonian (13) to the self energy $\Sigma(\vec{k}, \omega)$.

The results for the two branches are presented in Fig. 1 by Feynman diagrams and given by:

$$\begin{aligned} \Sigma^- (\vec{k}, \omega) = & \frac{1}{\hbar^2} \sum_{\vec{q}} \left[\left(u_{\vec{q}} \sigma_{\vec{k}} \alpha_{\vec{k}-\vec{q}} - v_{\vec{q}} \alpha_{\vec{k}} \sigma_{\vec{k}-\vec{q}} \right)^2 G_0^- (\vec{k} - \vec{q}, \omega - \omega_{\vec{q}}) \right. \\ & \left. + \left(u_{\vec{q}} \sigma_{\vec{k}} \beta_{\vec{k}-\vec{q}} + v_{\vec{q}} \gamma_{\vec{k}-\vec{q}} \alpha_{\vec{k}} \right)^2 G_0^+ (\vec{k} - \vec{q}, \omega - \omega_{\vec{q}}) \right]; \end{aligned} \quad (25a)$$

$$\begin{aligned} \Sigma^+ (\vec{k}, \omega) = & \frac{1}{\hbar^2} \sum_{\vec{q}} \left[\left(v_{\vec{q}} \beta_{\vec{k}} \gamma_{\vec{k}-\vec{q}} - u_{\vec{q}} \gamma_{\vec{k}} \beta_{\vec{k}-\vec{q}} \right)^2 G_0^+ (\vec{k} - \vec{q}, \omega - \omega_{\vec{q}}) \right. \\ & \left. + \left(u_{\vec{q}} \gamma_{\vec{k}} \alpha_{\vec{k}-\vec{q}} + v_{\vec{q}} \sigma_{\vec{k}-\vec{q}} \beta_{\vec{k}} \right)^2 G_0^- (\vec{k} - \vec{q}, \omega - \omega_{\vec{q}}) \right]. \end{aligned} \quad (25b)$$

These expressions are valid if the following two conditions are satisfied (with D the dimension):

$$\frac{1}{n_0 \xi^D} \left(\frac{U^{\uparrow\downarrow}}{U^{\uparrow\uparrow}} \right)^2 \ll 1; \quad (26a)$$

$$\frac{1}{n_0 \xi^D} \left(\frac{g}{\sqrt{n_0} U^{\uparrow\uparrow}} \right)^2 \ll 1. \quad (26b)$$

Condition (26a) shows that the average distance between the particles should be small as compared to the healing length and the background interactions where condition (26b) requires that the coupling to the molecule should be weak as compared to the condensate interactions. The latter condition implies that our approximation is only valid for narrow Feshbach resonances (small g).

If there is no coupling with the molecule ($g = 0$) the expressions (25) reduce to the well-known weak coupling result for the BEC-impurity Fröhlich polaron [17, 18]:

$$\Sigma^{pol} (\vec{k}, \omega) = \frac{n_0 (U^{\uparrow\downarrow})^2}{\hbar} \sum_{\vec{q}} \frac{E_{\vec{q}}^{\uparrow}}{\varepsilon_{\vec{q}}^{Bog}} \frac{1}{\hbar\omega - \hbar\omega_{\vec{q}} - E_{\vec{k}-\vec{q}}^{\downarrow} + i\delta}. \quad (27)$$

The physical states of the system correspond to the poles of the Green's function (24). Since the self energy is assumed small with respect to the mean field dispersion, the argument ω of the self energy can be replaced by the unperturbed energy $\varepsilon_{\vec{k}}^{\pm}$ for the determination of the poles of (24). This results in the following expression for the energies of the quasiparticles:

$$E_{\vec{k}}^{\pm} = \varepsilon_{\vec{k}}^{\pm} + \hbar \text{Re} \left[\Sigma^{\pm} (\vec{k}, \varepsilon_{\vec{k}}^{\pm}) \right]. \quad (28)$$

FIG. 1: The Feynman diagrams corresponding to the expressions (25) for the self energies of the two branches. The dashed lines denote a Bogoliubov excitation and the full lines are the unperturbed upper (+) and lower (-) quasiparticles.

The inverse of the imaginary part of the self-energy gives the lifetime of the quasiparticles τ_k^\pm :

$$\tau_k^\pm = \text{Im} \left[\Sigma_0 \left(\vec{k}, \varepsilon_k^\pm \right) \right]^{-1}. \quad (29)$$

We are also interested in the effective masses m^\pm which can be determined as:

$$\frac{1}{m^\pm} = \frac{1}{\hbar^2} \left. \frac{\partial^2 E_k^\pm}{\partial k^2} \right|_{k=0} \quad (30)$$

$$= \frac{1}{m^{MF(\pm)}} + \frac{1}{m^{Pert(\pm)}}. \quad (31)$$

where $m^{MF(\pm)}$ is the mean field effective mass (18) and the second term is the perturbative correction. Since this correction is assumed to be small we can write:

$$m^\pm \approx m^{MF(\pm)} \left(1 - \frac{m^{MF(\pm)}}{m^{Pert(\pm)}} \right). \quad (32)$$

A. Renormalization

A closer look at the arguments of the \vec{q} -summations in the expressions for the self energies (25) reveals a UV divergency if the dimension is larger than 1. This can be cured by using the renormalization equations (3). Since only the lowest order contribution of a perturbation series is considered, the renormalizing equations have to be considered to the same order. Inserting the renormalization equations (3) into the expressions for the mean field energies (17) and only considering the lowest order terms of an expansion in UR and g^2R results in:

$$\varepsilon_{\vec{k}}^- \rightarrow \varepsilon_{\vec{k}}^- + \frac{\sigma_{\vec{k}}^2}{\hbar^2} \sum_{\vec{q}} \frac{1}{\frac{\hbar^2 q^2}{2m_r} - E}; \quad (33a)$$

$$\varepsilon_{\vec{k}}^+ \rightarrow \varepsilon_{\vec{k}}^+ + \frac{\gamma_{\vec{k}}^2}{\hbar^2} \sum_{\vec{q}} \frac{1}{\frac{\hbar^2 q^2}{2m_r} - E}. \quad (33b)$$

Here, it is understood that the expressions on the right hand side are in terms of the physical coupling parameters $U_P^{\uparrow\downarrow}$, g_P and ε_P^M . Moreover, since the results for the self energies (25) are already of the order we are interested in the bare coupling parameters can be simply replaced by the physical ones in these expressions. Using (33) for the first term in the expression for the energies (28) exactly cancels the UV divergence.

VI. PHYSICAL SYSTEMS

The model is now applied to two distinct physical systems. First an impurity atom in a Bose-condensed atomic gas is considered in a three dimensional set-up where the impurity can form a molecule with one of the bosons. Then, a single spin down polariton in the presence of a Bose condensed gas of spin up polaritons is considered in two dimensions where a bipolariton can be formed from one spin up and one spin down polariton. In both cases losses are neglected and the systems are considered at equilibrium.

A. Ultracold atomic Gas system

The two-body s-wave scattering amplitude f_k for the scattering of a spin up and a spin down atom is related to the T -matrix (5) as

$$f_k = -\frac{m_r}{2\pi\hbar^2} T \left(\frac{\hbar^2 k^2}{2m_r} \right). \quad (34)$$

The inverse scattering amplitude is typically expanded for small k as:

$$-\frac{1}{f_k} = \frac{1}{a} + ik - \frac{1}{2} r_e k^2 + \mathcal{O}(k^4), \quad (35)$$

where a is the scattering length, which for the model is given by:

$$a = \frac{m_r}{2\pi\hbar^2} \left(U_P^{\uparrow\downarrow} - \frac{|g_P|^2}{\varepsilon} \right). \quad (36)$$

The effective range r_e is related to the Feshbach width R^* as $R^* = -r_e/2$, which for the model is:

$$R^* = R_{res}^* \left(1 - \frac{a_{bg}}{a}\right)^2, \quad (37)$$

with R_{res}^* the value at resonance:

$$R_{res}^* = \frac{\pi \hbar^4}{|g_P|^2 m_r^2 V}. \quad (38)$$

Note that since our approximation is only valid for small g_P , R_{res}^* should be large. The background scattering length a_{bg} is:

$$a_{bg} = \frac{m_r}{2\pi \hbar^2} U_P^{\uparrow\downarrow}. \quad (39)$$

Typically the scattering length in the vicinity of a magnetic Feshbach resonance is parametrized as:

$$a = a_{bg} \left(1 - \frac{\Delta B}{B - B_0}\right), \quad (40)$$

with B the magnetic field, B_0 the location of the resonance and ΔB the width, which are experimental parameters. In order to identify the scattering length of the model (36) with the physical scattering length (40) in the vicinity of the resonance an expansion of the molecular energy dependence on the magnetic field is performed:

$$\varepsilon(B) = \delta\mu (B - B_0) + \mathcal{O}((B - B_0)^2),$$

where $\delta\mu$ is the difference between the magnetic moment of the separated atoms and the magnetic moment of the molecular state. This identification shows that the model parameters can be expressed as a function of the experimental parameters as follows:

$$U_P^{\uparrow\downarrow} = \frac{2\pi \hbar^2 a_{bg}}{m_r}; \quad (41)$$

$$|g_P|^2 = U_P^{\uparrow\downarrow} \delta\mu \Delta B. \quad (42)$$

Note that ΔB needs to have the same sign as the background scattering length.

For the Bose gas the amplitude of the contact interactions is given by:

$$U^{\uparrow\uparrow} = \frac{4\pi \hbar^2 a_{BB}}{m_{\uparrow}}, \quad (43)$$

with a_{BB} the boson-boson scattering length. The two conditions (26) for the perturbative result to be valid can be written as a function of the experimental parameters:

$$2\pi^{3/2}\sqrt{n_0 a_{BB}^3} \left(\frac{a_{bg}}{a_{BB}}\right)^2 \left(\frac{m_\uparrow}{m_r}\right)^2 = \pi\alpha \left(\frac{m_\uparrow}{m_r}\right)^2 \ll 1 \quad (44a)$$

$$4\pi^{3/2}\sqrt{n_0 a_{BB}^3} \frac{m_\uparrow}{m_r} \frac{a_{bg}}{a_{BB}} \frac{\delta\mu\Delta B}{n_0 U^{\uparrow\uparrow}} \ll 1. \quad (44b)$$

where the first condition was also expressed as a function of the dimensionless polaronic coupling parameter $\alpha = a_{bg}^2 / (a_{BB}\xi)$ [5]. This shows that the first condition (44a) expresses the diluteness condition $n_0 a_{BB}^3 \ll 1$, weak background interactions and that the masses should not be too different. The second condition expresses the same and additionally that the width of the resonance ΔB should be small, corresponding to a narrow Feshbach resonance.

Since ultralow temperatures are considered the limit $E \rightarrow 0$ is taken in the renormalization expressions (3).

B. Polaritonic system

If the exciton-photon coupling is sufficiently strong and the temperature sufficiently low only the lower polaritons are populated of which the dispersion is given by [9]:

$$E_{\vec{k}} = \frac{1}{2} \left(\delta - \sqrt{\delta^2 + \Omega_R^2} \right) + \frac{k^2}{2m_P}. \quad (45)$$

Here Ω_R is the Rabi frequency, δ is the detuning and m_P is the effective mass of the lower polaritons:

$$m_P = \frac{m_{cav}}{\sin^2 \theta}. \quad (46)$$

With m_{cav} the effective photon mass and $\sin \theta$ the photonic Hopfield coefficient (see later). The interaction parameters depend on the excitonic Hopfield coefficient $\cos \theta$ as:

$$U^{\uparrow\uparrow} = V \cos^4 \theta; \quad (47)$$

$$U_P^{\uparrow\downarrow} = V_{bg} \cos^4 \theta; \quad (48)$$

$$g_P = V_{BX} \cos^2 \theta. \quad (49)$$

Where V is the interaction amplitude between the spin up excitons, V_{bg} is the background interaction amplitude between spin up and spin down excitons and V_{BX} denotes the coupling of the excitons to the biexciton. For the Hopfield factors the dependence on the wave vector

is neglected and they are given by:

$$\cos \theta = \sqrt{\frac{1}{2} \left(1 + \frac{\delta}{\sqrt{\delta^2 + \hbar^2 \Omega_R^2}} \right)}; \quad (50)$$

$$\sin \theta = \sqrt{\frac{1}{2} \left(1 - \frac{\delta}{\sqrt{\delta^2 + \hbar^2 \Omega_R^2}} \right)}. \quad (51)$$

The validity conditions (26) can for this system be written as

$$\frac{\cos^4 \theta}{\sin^2 \theta} \frac{m_{cav}}{\hbar^2} \frac{V_{bg}^2}{V} \ll 1; \quad (52a)$$

$$\frac{1}{\sin^2 \theta} \frac{m_{cav}}{\hbar^2} \frac{V_{BX}^2}{n_0 V} \ll 1. \quad (52b)$$

Which again shows that the background interactions and the coupling to the biexciton should be weak and the Hopfield factor $\sin \theta$ may not be too small.

We are interested in the behavior of the T -matrix at low temperature and thus small scattering energy E . In 3D this amounted to simply taking the limit $E \rightarrow 0$. However in 2D this limit results in a logarithmic IR divergence. Fortunately, in 2 dimensions the T -matrix depends only logarithmically on the energy [19], so that we can set $E = -\mu_\uparrow$ in the renormalizing expressions (3).

VII. RESULTS AND DISCUSSION

In the following we start by summarizing some analytical results for the background (vanishing coupling to the molecule) and show how the perturbative single-channel results are obtained. Then the results are examined for the specific systems of a ^6Li impurity in a Na condensate and for the polaritonic system in a GaAs-based microcavity.

A. Background results and perturbative single channel-model

If the coupling to the molecule vanishes ($g = 0$), the \vec{k} -summation in the expression for the self energy (27) can be performed analytically in some cases. We start by summarizing these results. We also indicate how perturbative single-channel results with the effective interaction strength (6) are obtained for the different systems.

1. Ultracold atomic Gas system

If there is no coupling to the molecule ($g = 0$) the real part of the impurity self energy (27) at $k = 0$ is given by:

$$\frac{\text{Re} [\hbar \Sigma^{bg}(0)]}{\hbar^2 / (m_\uparrow \xi^2)} = \frac{1}{2\pi^2} \frac{1}{n_0 \xi^3} \left(\frac{a_{bg}}{a_{BB}} \right)^2 \frac{1 + \frac{m_\uparrow}{m_\downarrow}}{1 - \frac{m_\downarrow}{m_\uparrow}} \left(1 + \frac{\left(\frac{m_\downarrow}{m_\uparrow} \right)^2}{\sqrt{1 - \left(\frac{m_\downarrow}{m_\uparrow} \right)^2}} \ln \left[\frac{m_\uparrow - \sqrt{m_\uparrow^2 - m_\downarrow^2}}{m_\downarrow} \right] \right). \quad (53)$$

The imaginary part of the self energy is only non-zero if $k > k_c = m_\downarrow / (m_\uparrow \xi)$. This corresponds to the Landau criterion for superfluidity [20] which states that energy can only be dissipated to the condensate if $k > k_c$. In this case the imaginary part is given by:

$$\frac{\text{Im} \Sigma^{bg}(k > k_c)}{\hbar / (m_\uparrow \xi^2)} = -\frac{1}{16\pi} \frac{1}{n_0 \xi^3} \frac{1}{\xi k} \left(\frac{a_{bg}}{a_{BB}} \right)^2 \frac{(m_\downarrow + m_\uparrow)^2}{m_\downarrow m_\uparrow} \left(\frac{Q_c}{2} \sqrt{4 + Q_c^2} - 2 \sinh^{-1} \left[\frac{Q_c}{2} \right] \right); \quad (54)$$

with:

$$Q_c = \frac{2 \frac{m_\downarrow}{m_\uparrow} \sqrt{1 - \left(\frac{m_\downarrow}{m_\uparrow} \right)^2} + \xi^2 k^2 - 2\xi k}{\left(\frac{m_\downarrow}{m_\uparrow} \right)^2 - 1}. \quad (55)$$

The effective mass m_\downarrow^* is given by:

$$\frac{m_\downarrow}{m_\downarrow^*} = 1 + \frac{1}{6\pi^2} \frac{1}{n_0 \xi^3} \left(\frac{a_{bg}}{a_{BB}} \right)^2 \frac{1}{\left(1 - \frac{m_\downarrow}{m_\uparrow} \right)^2} \left(3 + \frac{2 + \left(\frac{m_\downarrow}{m_\uparrow} \right)^2}{\sqrt{1 - \left(\frac{m_\downarrow}{m_\uparrow} \right)^2}} \ln \left[\frac{m_\uparrow - \sqrt{m_\uparrow^2 - m_\downarrow^2}}{m_\downarrow} \right] \right) \quad (56)$$

The perturbative single-channel model corresponds to these background results with the replacement of the background scattering length a_{bg} with the Feshbach enhanced scattering length (40).

2. Polaritonic system

If there is no coupling to the molecule ($g = 0$) the real part of the self energy at $k = 0$ becomes:

$$\text{Re} [\hbar \Sigma^{bg}(0)] = \frac{n_0 V_{bg}^2 m_{cav} \cos^8 \theta}{4\pi \hbar^2 \sin^2 \theta}. \quad (57)$$

The effective mass in this case is given by:

$$\frac{1}{m^*} = \frac{\sin^2 \theta}{m_{cav}} - \frac{V_{bg}^2}{4\pi \hbar^2 V} \cos^4 \theta. \quad (58)$$

For the imaginary part the integral has to be done numerically. Similar as for the ultracold atomic system, the lifetime is infinite for $k < k_c$, with k_c the Landau critical value. For $k > k_c$ energy can be dissipated to the condensate by emitting a Bogoliubov excitation which results in a finite lifetime.

In this case the perturbative single-channel model corresponds to these background results with the following substitution for the background interaction amplitude V_{bg} :

$$V_{bg} \rightarrow V_{bg} + \frac{|g|^2}{2E_0 + \varepsilon^M}. \quad (59)$$

B. Ultracold atomic Gas system

The results are now studied for a ^6Li impurity in a Na condensate, corresponding to the parameters $m_\downarrow/m_\uparrow = 0.263$ and $a_{bg}/a_{BB} = -1.336$ [8, 21]. In Ref. [8] various narrow interspecies Feshbach resonances are determined for this system. We will not focus on a specific resonance but examine the typical expected behavior. The difference in magnetic moment $\delta\mu$ is typically of the order of the Bohr magneton: $\delta\mu = e\hbar/(2m_e)$, with e the elementary charge and m_e the electron mass. A coupled-channel calculation in Ref. [8] revealed that the widths of the resonances ΔB are of the order of mG so we take for the width: $\Delta B = -1\text{mG}$. For the Na condensate the typical density $n_0 = 10^{14} \text{ cm}^{-3}$ is considered. With these parameters we find:

$$2\pi^{3/2} (n_0 a_{BB}^3)^{1/2} \left(\frac{a_{bg}}{a_{BB}} \right)^2 \left(\frac{m_\uparrow}{m_r} \right)^2 \approx 0.25; \quad (60)$$

$$4\pi^{3/2} (n_0 a_{BB}^3)^{1/2} \frac{m_\uparrow}{m_r} \frac{a_{bg}}{a_{BB}} \frac{\delta\mu \Delta B}{n_0 U^{\uparrow\uparrow}} \approx 0.067. \quad (61)$$

This shows that the conditions (44) are satisfied and the system is in the perturbative regime.

For the units the condensate chemical potential $\mu^\uparrow = n_0 U^{\uparrow\uparrow}$, the corresponding healing length ξ and the boson mass m_\uparrow are used. In these units the self energy exhibits a $(n_0 \xi^3)^{-1}$ dependence on the density and in the figures below the self energy is always multiplied with this factor which for the considered system is $n_0 \xi^3 \approx 14.5$.

In Fig. 2 the dispersions of the mean field quasiparticles (17) are presented as a function of the applied magnetic field B at $k = 0$. Note that due to the presence of the condensate the resonance is shifted from B_0 to

$$B_0^* = B_0 + \frac{n_0}{\delta\mu} \left(\frac{2\pi\hbar^2 a_{bg}}{m_r} + \frac{4\pi\hbar^2 a_{BB}}{m_\uparrow} \right). \quad (62)$$

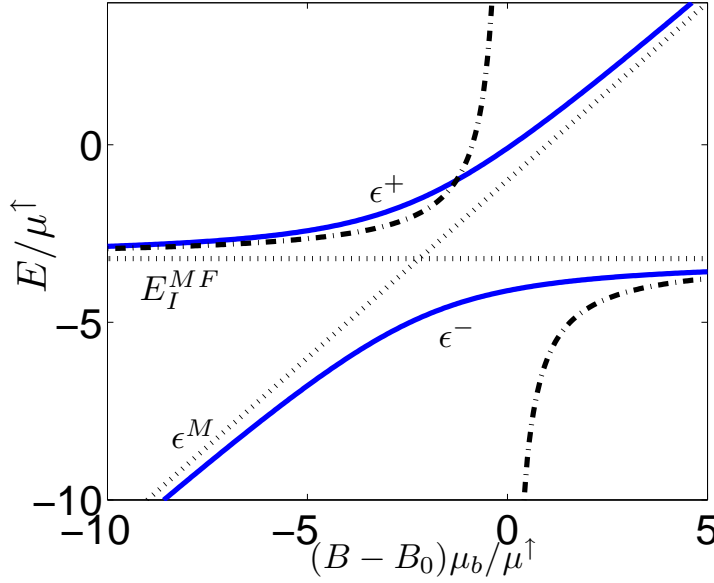


FIG. 2: The quasiparticle dispersions ε^\pm at the mean field level (17) due to the mixing of the energy levels of the molecule and the ^6Li impurity in the presence of a Na condensate as a function of the magnetic field B at $k = 0$. The dotted lines are the energies of the impurity and the molecule which are shifted due to the presence of the condensate: $E_I^{MF} = n_0 U_P^{\uparrow\downarrow}$ and $\varepsilon^M = -\varepsilon_0^M - n_0 U^{\uparrow\uparrow}$. The dash-dotted line gives the single-channel result.

Which for the system under consideration gives $(B_0^* - B_0) \mu_b / \mu^\uparrow \approx -2.21$. Also the result of the perturbative single-channel calculation is presented, which strongly deviates from the perturbative result around the resonance and diverges at $B = B_0$. The single-channel result provides a good approximation far from the resonance for the branch with energy closest to the impurity energy (which we will denote as the impurity-like branch). This could be expected since in this case the occupation of the closed channel is small.

In Fig. 3 (a) the perturbative corrections $\Delta E^\pm = \text{Re} \left[\hbar \Sigma^\pm \left(0, \varepsilon_k^\pm \right) \right]$ to the mean field energies, as defined in Eq. (28), are presented as a function of the magnetic field B at $k = 0$. The background result (53) and the perturbative single-channel result are also shown. Again the perturbative single channel result diverges at $B = B_0$ and is only a good approximation for the impurity-like branch far from the resonance. In Fig. 3 (b) the corresponding lifetime (29) of the upper branch is presented. The lifetime is infinite in the limit $B \rightarrow -\infty$ (impurity-like) and zero for $B \rightarrow \infty$ (molecular-like). In the vicinity of the

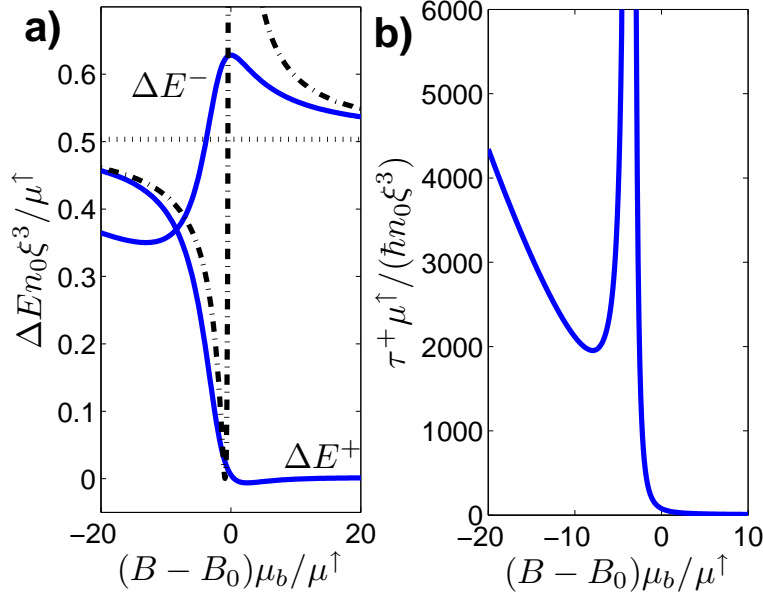


FIG. 3: a) The perturbative correction ΔE^\pm to the mean field quasiparticle energies from Fig. 2 as a function of the magnetic field B at $k=0$ for a ^6Li impurity in a Na condensate. The dashed line is the background result (53) and the dot-dashed line is the perturbative single-channel result. b) The corresponding lifetime of the upper branch.

resonance a strong peak is observed with a maximum corresponding to an infinite lifetime at $(B - B_0) \mu_b / \mu^\uparrow \approx -3.55$.

The lifetimes of the two branches are presented in Fig. 4 as a function of the wave number k at $B = B_0$. Also the lifetime in the absence of a molecule is presented (the inverse of (54)) which is only finite for $k > k_c$, with k_c the Landau critical value, as discussed before. For the lower branch a similar behavior is observed with an infinite lifetime at small k until the Landau critical value k_c^- is reached. This value is the lowest possible value of k that satisfies conservation of energy and momentum for the emission of a Bogoliubov excitation and is thus determined by:

$$k_c^\pm = \min \left\{ k = |\vec{k}| : \exists \vec{q} \mid \varepsilon_k^\pm = \varepsilon_{k-\vec{q}}^\pm + \varepsilon_{\vec{q}}^{Bog} \right\}. \quad (63)$$

For the upper branch a finite lifetime is also found at small k since it can also decay to the lower branch with the emission of a Bogoliubov excitation. Once k reaches the Landau critical value k_c^+ , as determined by Eq. (63), an extra decay channel opens corresponding to the emission of a Bogoliubov excitation.

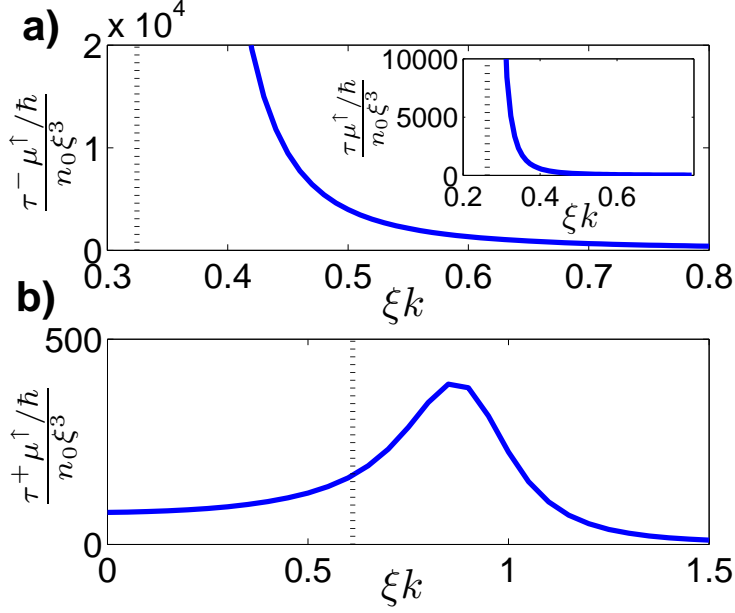


FIG. 4: The lifetimes of the lower (a) and the upper (b) quasiparticle branches are presented as a function of the wave number k for a ^6Li impurity in a Na condensate. The inset in (a) shows the lifetime in the absence of a molecule. The dashed lines indicate the Landau critical values.

In Fig. 5 (a) the mean field effective masses (18) are presented for the two quasiparticle branches and in (b) the perturbative correction $1/m^{\text{Pert}(\pm)}$ is shown (as defined in (30)). The background result and the perturbative single-channel result are also depicted. Again we find that the single-channel result strongly deviates from the perturbative result in the vicinity of the resonance but is a reasonable approximation for the impurity-like branch far from the resonance.

C. Polaritonic system

The system parameters for a GaAs-based microcavity are introduced [12, 22, 23], with $\Omega_R = 3.26$ meV for the Rabi frequency, $E_{bix} = 3$ meV for the molecular binding energy and $m_{cav} = 2.62 \times 10^{-5} m_e$ for the effective photon mass, with m_e the electron mass. Only considering the dominant Coulomb interactions for the spin up lower polaritons and neglecting the contribution due to phase space philling results in: $V = 3a_B e^2 / \varepsilon \approx 3.73 \times 10^{-11}$ meV cm² (with a_B the exciton Bohr radius, e the electron charge and ε the dielectric constant) [24]. The ratio of the direct interactions is approximated with a density independent constant:

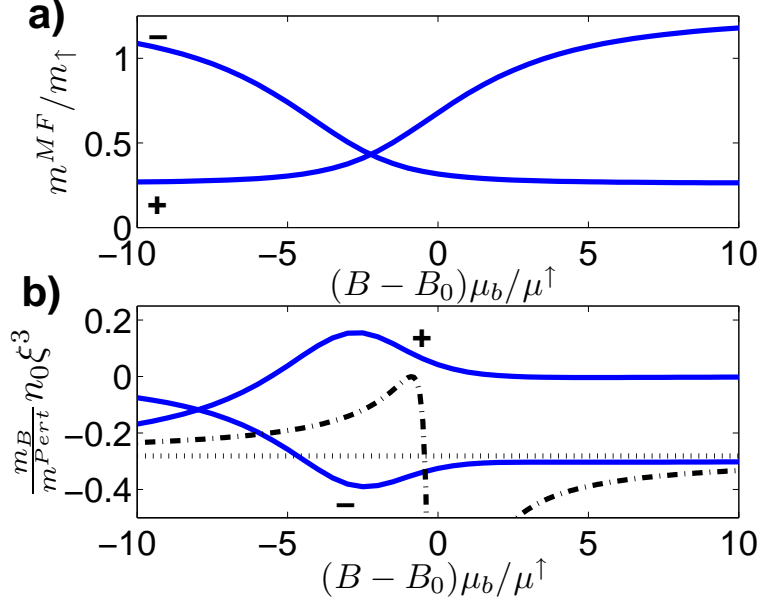


FIG. 5: The effective masses are presented for the two branches as a function of the magnetic field B for a ${}^6\text{Li}$ impurity in a Na condensate. In (a) the mean field result (18) is presented and in (b) the perturbative correction to the inverse effective mass is shown. The dotted line is the background result (56) and the dash-dotted line is the perturbative single-channel result.

$V_{bg}/V = -0.39$ and for the polariton condensate density $n_0 = 10^{10} \text{ /cm}^2$ is considered. For the coupling strength to the biexciton $V_{BX} = 3.73 \times 10^{-6} \text{ meV cm}$ is taken. The biexciton mass is taken infinite since it is much larger than the effective photon mass. With these parameters we find:

$$\frac{\cos^4 \theta}{\sin^2 \theta} \frac{m_{cav}}{\hbar^2} \frac{V_{bg}^2}{V} \approx 7.56 \times 10^{-6} \frac{\cos^4 \theta}{\sin^2 \theta}; \quad (64)$$

$$\frac{1}{\sin^2 \theta} \frac{m_{cav}}{\hbar^2} \frac{V_{BX}^2}{n_0 V} \approx 3.25 \times 10^{-5} \frac{1}{\sin^2 \theta}. \quad (65)$$

This shows that the validity conditions (52) are well-satisfied, except at very small values of $\sin \theta$ (or large positive values for the detuning δ), which correspond to an almost pure exciton with large mass (46).

For the units the condensate interaction energy at zero detuning $E_0 = Vn_0/4$, the corresponding healing length $\xi = \hbar/\sqrt{2m_{cav}E_0}$ and twice the photon mass $2m_{cav}$ are used. In these units the self energy exhibits a $(n_0\xi^2)^{-1}$ dependence on the density and we will multiply the self energy with $n_0\xi^2$ for the results in this section. Note that the factor $n_0\xi^2$ can

be quite large and for the considered system is $n_0\xi^2 \approx 1560$.

In Fig. 6 the mean field quasiparticles dispersions (17) are presented as a function of the detuning for $k = 0$. The resonance is shifted from the two-body result due to the presence of the condensate to δ_R , determined by:

$$V_{bg}n_0 \cos^4 \theta = -\varepsilon_0^M - Vn_0 \cos^4 \theta + \left(\delta_R - \sqrt{\delta_R^2 + \hbar^2 \Omega_R^2} \right); \quad (66)$$

which for the considered system is $\delta_R \approx 2.17E_0$. The manifestation of these two branches has been observed experimentally for this system in Ref. [12]. Since both the molecular and the lower polariton energies remain finite in the limit $\delta \rightarrow \infty$, the quasiparticles remain superpositions of bound and scattering states. Also the result from a single channel calculation is presented in Fig. 6 which results in a reasonable approximation for the quasiparticle branch that lies close to the lower polariton far from the resonance.

In Fig. 7 (a) the perturbative corrections $\Delta E^\pm = \text{Re} [\hbar \Sigma^\pm(0, \varepsilon_0^\pm)]$, as defined in Eq. (28), to the mean field quasiparticle energies from Fig. 6 are presented as a function of the detuning. In order to focus on the effect of the bipolariton the results are divided by the background result (57) which is presented in the inset. Also the perturbative single-channel result is shown which is a good approximation for the correction to the lower branch at large negative δ . It is a poorer approximation of the upper branch at large positive δ since the quasiparticle remains a superposition in this limit, as was clear from Fig. 6. In Fig. 7 (b) the corresponding lifetime is presented for the upper branch. Also here a peak is observed in the vicinity of the resonance but now with a finite height. Note furthermore that in contrast with the ultracold atomic system a finite lifetime is found for $\delta \rightarrow \infty$ ("impurity-like") which is due to the quasiparticle remaining a superposition of molecular and impurity states and an infinite lifetime is found for $\delta \rightarrow -\infty$ ("molecular-like").

The lifetimes of the two branches as a function of the wave number k are presented in Fig. 8 and the same qualitative behavior is observed as for the ultracold atomic gas system in Fig. 4. The background lifetime (shown in the inset of Fig. 8 (a)) and the lower branch lifetime are infinite for $k < k_c$, with k_c the Landau critical value determined by Eq. (63), since it is impossible to dissipate energy to the condensate. The lifetime of the upper branch is also finite at small k since it can decay to the lower branch with the emission of a Bogoliubov excitation. Once $k > k_c$ the possibility to emit a Bogoliubov excitation gives an extra contribution to the (inverse) lifetime.

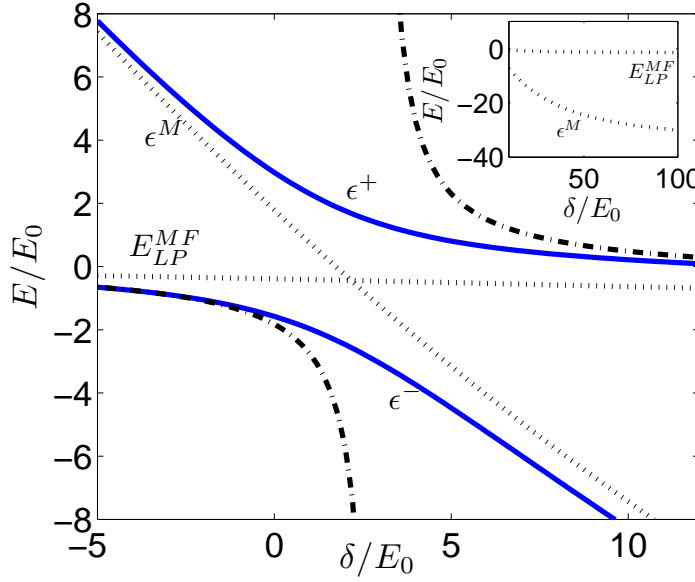


FIG. 6: The mean field result for the quasiparticle energies (17) as a function of the detuning δ for a spin down lower polariton in the presence of a Bose-condensed gas of spin up lower polaritons in a GaAs-based microcavity. The dotted lines are the mean field energies with a shift due to the presence of the condensate of the spin down lower polariton and the bipolariton: $E_{LP}^{MF} = V_{bg}n_0 \cos^4 \theta$ and $\epsilon^M = -\epsilon_0^M - Vn_0 \cos^4 \theta - \left(\delta - \sqrt{\delta^2 + \Omega_R^2} \right)$ (The inset shows these values at relatively large δ , showing that both converge to a finite value for $\delta \rightarrow \infty$). The dash-dotted line gives the result from the perturbative single-channel calculation.

The mean field result for the effective masses as a function of the detuning δ are presented in Fig. 9 (a) and the perturbative correction in (b). The background contribution (58) is also indicated as well as the result from the perturbative single-channel calculation. The single-channel result is again a good approximation for the lower branch for large negative δ while for large positive δ there remains a discrepancy since the quasiparticles remain a superposition in the limit $\delta \rightarrow \infty$.

VIII. CONCLUSIONS

A coupled-channel calculation is presented for the polaronic system consisting of an impurity in a Bose-condensed gas in the presence of an interspecies Feshbach resonance. At

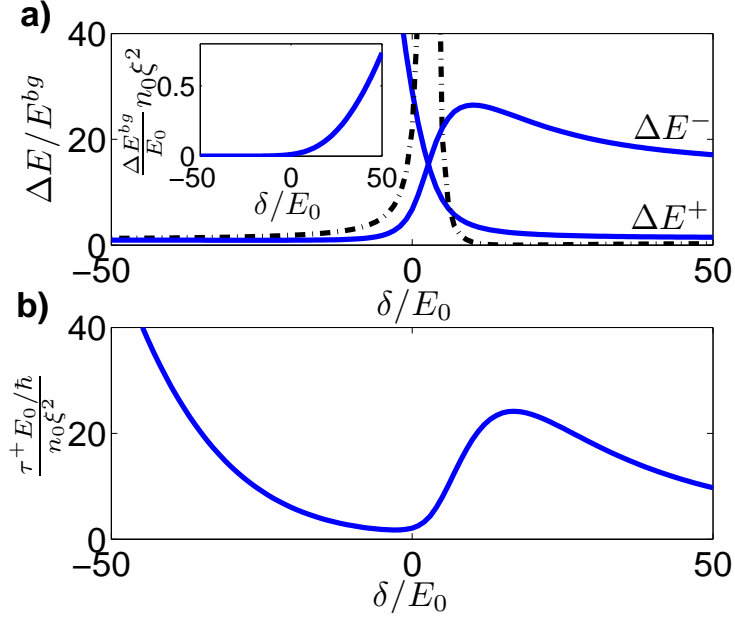


FIG. 7: a) The perturbative corrections ΔE^\pm to the mean field energies divided by the background result (57) (shown in the inset) are presented as a function of the detuning δ for a spin down lower polariton in the presence of a Bose-condensed gas of spin up lower polaritons in a GaAs-based microcavity. The dash-dotted line is the result from a perturbative single-channel calculation. b) The corresponding lifetime for the upper branch.

the mean field level, two quasiparticles branches are found that consist of a mixing of the impurity and the molecular states. Due to the presence of the condensate, the location of the Feshbach resonance is shifted. The corrections to the mean field result are examined within perturbation theory. This approximation is valid if both the coupling to the molecular channel and the background interactions are weak with respect to the condensate interaction energy and if the condensate healing length is sufficiently large compared to the average interparticle distance.

The model is applied to two physical systems by identifying the low energy T -matrix and using a proper renormalization scheme. This allows to express the model parameters as a function of the physical parameters. The systems under consideration are a ^6Li impurity atom in a Bose-condensed atomic Na gas in 3 dimensions and a spin down lower polariton in the presence of a Bose-Einstein condensate of spin up lower polaritons in a GaAs-based cavity in two dimensions. These systems are both in the regime where perturbation theory

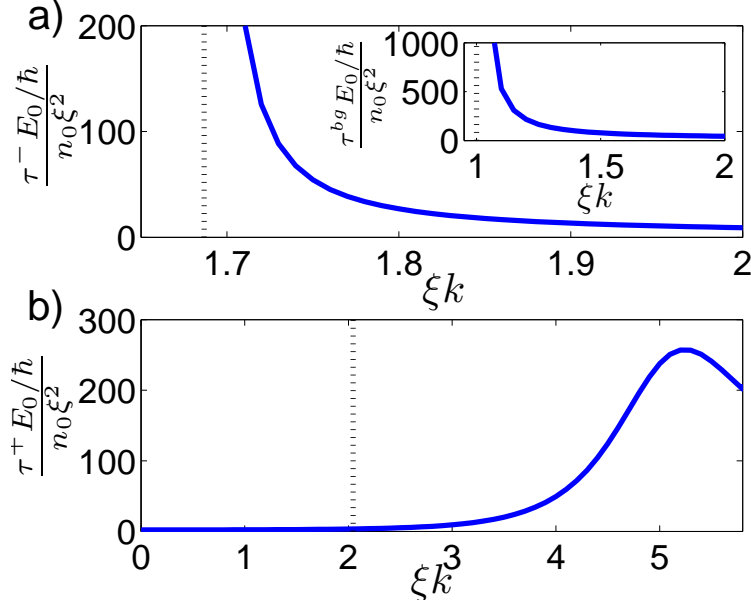


FIG. 8: The lifetime of the lower (a) and upper (b) quasiparticle branches are presented as a function of the wave number k at zero detuning ($\delta = 0$) for a spin down lower polariton in the presence of a Bose-condensed gas of spin up lower polaritons in a GaAs-based microcavity. The inset in (a) presents the background contribution of the lower polariton in the absence of a bipolariton. The dotted lines indicate the Landau critical values, as determined by Eq. (63).

is valid.

Far from the resonance the results for the quasiparticle branch with an energy close to the impurity energy are well-reproduced with a perturbative single-channel calculation. However, in the vicinity of the resonance the results from the two calculations strongly deviate since the perturbative single-channel calculation diverges at the resonance while the coupled-channel calculation predicts finite results.

Some important properties of the quasiparticles, such as the lifetime and the effective mass were examined. The lifetime of the lower branch is infinite at small wave number k and becomes finite once k exceeds the Landau critical value which means it can dissipate energy to the condensate. The lifetime of the upper branch is also finite at small k since it can decay to the lower branch with the emission of a Bogoliubov excitation. For k larger than the Landau critical value it can also emit a Bogoliubov excitation which results in two contributions to the (inverse) lifetime.

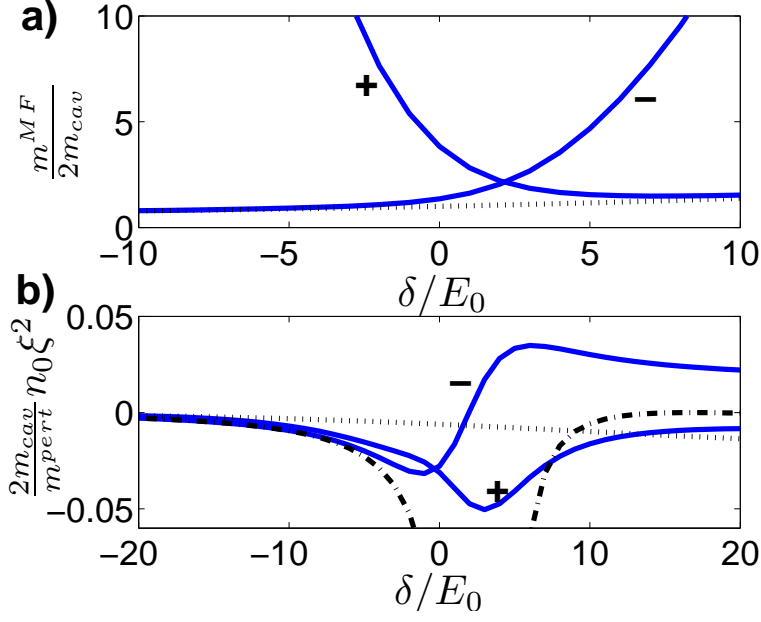


FIG. 9: a) The mean field masses of the lower (-) and upper (+) quasiparticle branches (18) as a function of the detuning δ for a spin down lower polariton in the presence of a Bose-condensed gas of spin up lower polaritons in a GaAs-based microcavity. The dotted line presents the lower polariton mass. b) The corresponding perturbative corrections to the mean field masses. The dotted line is the background result (58) and the dash-dotted line gives the result from the perturbative single-channel calculation.

In general the perturbative results are found to be very weak as compared to the mean field results for the considered systems, in particular for the polariton system. However, for cold atomic systems, where a wide range in Feshbach resonance widths exists, the corrections to mean field can be made appreciable.

Appendix A: Calculation of the self energy

If the interaction representation is used the Green's function (22) can generally be written as a perturbative expansion in the interaction Hamiltonian \hat{H}_I as [16]:

$$G(\vec{K}, t - t') = \frac{-i}{\langle 0 | U(\infty, -\infty) | 0 \rangle} \sum_{n=0}^{\infty} \frac{1}{n! (i\hbar)^n} \times \int_{-\infty}^{\infty} dt_1 \dots dt_n \langle 0 | \mathcal{T} [\hat{\Psi}_{\vec{K}}(t) \hat{H}_I(t_1) \dots \hat{H}_I(t_n) \hat{\Psi}_{\vec{K}}^\dagger(t')] | 0 \rangle, \quad (\text{A1})$$

where $U(t, t_0)$ denotes the time evolution operator:

$$U(t, t_0) = \mathcal{T} \exp \left[\frac{1}{i\hbar} \int_{t_0}^t dt_1 \hat{H}_I(t_1) \right]. \quad (\text{A2})$$

The effect of the denominator $\langle 0 | U(\infty, -\infty) | 0 \rangle$ is to cancel all contributions from disconnected diagrams from the series.

For the considered model system with the interaction Hamiltonian (19) the first order correction is zero since it involves the expectation value of a single creation or annihilation operator with respect to the vacuum. We are interested in the lowest non-vanishing correction which means we have to calculate the following expectation value:

$$\left\langle 0 \left| \mathcal{T} \left[\hat{\Psi}_{\vec{k}}(t) \hat{H}_I(t_1) \hat{H}_I(t_2) \hat{\Psi}_{\vec{k}}^\dagger(t') \right] \right| 0 \right\rangle. \quad (\text{A3})$$

1. Expectation values of the quasiparticle operators

All the needed expectation values are of the following form:

$$\left\langle 0 \left| \mathcal{T} \left[\hat{\Psi}_{\vec{K}}(t) \left(\gamma_{\vec{k}_1+\vec{q}_1} \hat{\Phi}_{\vec{k}_1+\vec{q}_1}^\dagger(t_1) + \sigma_{\vec{k}_1+\vec{q}_1} \hat{\Psi}_{\vec{k}_1+\vec{q}_1}^\dagger(t_1) \right) \left(\beta_{\vec{k}_1} \hat{\Phi}_{\vec{k}_1}(t_1) - \alpha_{\vec{k}_1} \hat{\Psi}_{\vec{k}_1}(t_1) \right) \right. \right. \right. \\ \left. \left. \times \left(\gamma_{\vec{k}_2+\vec{q}_2} \hat{\Phi}_{\vec{k}_2+\vec{q}_2}^\dagger(t_2) + \sigma_{\vec{k}_2+\vec{q}_2} \hat{\Psi}_{\vec{k}_2+\vec{q}_2}^\dagger(t_2) \right) \left(\beta_{\vec{k}_2} \hat{\Phi}_{\vec{k}_2}(t_2) - \alpha_{\vec{k}_2} \hat{\Psi}_{\vec{k}_2}(t_2) \right) \hat{\Psi}_{\vec{K}}^\dagger(t') \right] \right| 0 \right\rangle \quad (\text{A4})$$

First the contribution with six creation or annihilation operators for the lower branch is considered:

$$\sigma_{\vec{k}_1+\vec{q}_1} \alpha_{\vec{k}_1} \sigma_{\vec{k}_2+\vec{q}_2} \alpha_{\vec{k}_2} \left\langle 0 \left| \mathcal{T} \left[\hat{\Psi}_{\vec{K}}(t) \hat{\Psi}_{\vec{k}_1+\vec{q}_1}^\dagger(t_1) \hat{\Psi}_{\vec{k}_1}(t_1) \hat{\Psi}_{\vec{k}_2+\vec{q}_2}^\dagger(t_2) \hat{\Psi}_{\vec{k}_2}(t_2) \hat{\Psi}_{\vec{K}}^\dagger(t') \right] \right| 0 \right\rangle. \quad (\text{A5})$$

This can be rewritten with Wick's theorem. We will only consider connected diagrams since the disconnected cancel with the denominator in (A1) and because we are interested in a single particle at zero temperature, the expectation value of the occupation number is zero. This means only two contributions remain:

$$\sigma_{\vec{k}_1+\vec{q}_1} \alpha_{\vec{k}_1} \sigma_{\vec{k}_2+\vec{q}_2} \alpha_{\vec{k}_2} \left\langle 0 \left| \mathcal{T} \left[\hat{\Psi}_{\vec{K}}(t) \hat{\Psi}_{\vec{k}_1+\vec{q}_1}^\dagger(t_1) \hat{\Psi}_{\vec{k}_1}(t_1) \hat{\Psi}_{\vec{k}_2+\vec{q}_2}^\dagger(t_2) \hat{\Psi}_{\vec{k}_2}(t_2) \hat{\Psi}_{\vec{K}}^\dagger(t') \right] \right| 0 \right\rangle \\ \rightarrow \sigma_{\vec{K}} \alpha_{\vec{K}} \alpha_{\vec{k}_1} \sigma_{\vec{k}_1} iG_0^-(\vec{K}, t - t_1) iG_0^-(\vec{k}_1, t_1 - t_2) iG_0^-(\vec{K}, t_2 - t') \delta_{\vec{K}, \vec{k}_1+\vec{q}_1} \delta_{\vec{k}_1, \vec{k}_2+\vec{q}_2} \delta_{\vec{k}_2, \vec{K}} \\ + \alpha_{\vec{K}} \sigma_{\vec{K}} \sigma_{\vec{k}_2} \alpha_{\vec{k}_2} iG_0^-(\vec{K}, t - t_2) iG_0^-(\vec{k}_2, t_2 - t_1) iG_0^-(\vec{K}, t_1 - t') \delta_{\vec{K}, \vec{k}_2+\vec{q}_2} \delta_{\vec{k}_2, \vec{k}_1+\vec{q}_1} \delta_{\vec{k}_1, \vec{K}}, \quad (\text{A6})$$

with $G_0^\pm(\vec{q}, \omega)$ the Green's function of the unperturbed quasiparticles which in the frequency representation is given by:

$$G_0^\pm(\vec{q}, \omega) = \frac{\hbar}{\hbar\omega - \varepsilon_{\vec{k}}^\pm + i\delta}, \quad (\text{A7})$$

with $\varepsilon_{\vec{k}}^\pm$ the mean field quasiparticle dispersions (17). There are four terms with four creation or annihilation operators for the lower branch for which Wick's theorem can be used again. Again we only consider the connected diagrams for a single particle at zero temperature which results in the following two terms:

$$\begin{aligned} & \sigma_{\vec{K}} \beta_{\vec{k}_1} \gamma_{\vec{k}_1} \alpha_{\vec{K}} iG_0^-(\vec{K}, t - t_1) iG_0^-(\vec{K}, t_2 - t') iG_0^+(\vec{k}_1, t_1 - t_2) \delta_{\vec{K}, \vec{k}_1 + \vec{q}_1} \delta_{\vec{k}_2, \vec{K}} \delta_{\vec{k}_1, \vec{k}_2 + \vec{q}_2} \\ & + \gamma_{\vec{k}_2} \alpha_{\vec{K}} \sigma_{\vec{K}} \beta_{\vec{k}_2} iG_0^-(\vec{K}, t - t_2) iG_0^-(\vec{K}, t_1 - t') iG_0^+(\vec{k}_2, t_2 - t_1) \delta_{\vec{K}, \vec{k}_2 + \vec{q}_2} \delta_{\vec{k}_1, \vec{K}} \delta_{\vec{k}_1 + \vec{q}_1, \vec{k}_2} \end{aligned} \quad (\text{A8})$$

Note that the term with only two creation and annihilation operators of the lower branch represents a disconnected diagram and thus also cancels with the denominator of (A1), which means we have all the different contributions for the expectation value (A4).

2. Expectation values of the Bogoliubov operators

Typically expectation values of the following form are needed:

$$\begin{aligned} & \left\langle 0 \left| \mathcal{T} \left[\left(u_{\vec{q}_1} \hat{\alpha}_{\vec{q}_1}(t_1) - v_{\vec{q}_1} \hat{\alpha}_{-\vec{q}_1}^\dagger(t_1) \right) \left(u_{\vec{q}_2} \hat{\alpha}_{\vec{q}_2}(t_2) - v_{\vec{q}_2} \hat{\alpha}_{-\vec{q}_2}^\dagger(t_2) \right) \right] \right| 0 \right\rangle \\ & = -u_{\vec{q}_1} v_{\vec{q}_1} [iD(\vec{q}_1, t_1 - t_2) + iD(-\vec{q}_1, t_2 - t_1)] \delta_{\vec{q}_1, -\vec{q}_2}. \end{aligned} \quad (\text{A9})$$

Here $D(\vec{q}, t_1 - t_2)$ denotes the unperturbed Green's function of the Bogoliubov excitations which in frequency representation is given by:

$$D(\vec{q}, \omega) = \frac{\hbar}{\hbar\omega - \varepsilon_{\vec{k}}^{Bog} + i\delta}, \quad (\text{A10})$$

with $\varepsilon_{\vec{k}}^{Bog}$ the Bogoliubov dispersion (9).

3. Expression for the self energy

Bringing everything together gives the following result:

$$\begin{aligned}
& \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{\infty} dt_2 \left\langle 0 \left| \mathcal{T} \left[\widehat{\Psi}_{\vec{K}}(t) \widehat{H}_I(t_1) \widehat{H}_I(t_2) \widehat{\Psi}_{\vec{K}}^\dagger(t') \right] \right| 0 \right\rangle \\
&= 2 \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{\infty} dt_2 \sum_{\vec{q}} G_0^- \left(\vec{K}, t - t_1 \right) G_0^- \left(\vec{K}, t_2 - t \right) \\
&\times \left\{ \left[\left(u_{\vec{q}} \sigma_{\vec{K}} \alpha_{\vec{K}-\vec{q}} - v_{\vec{q}} \alpha_{\vec{K}} \sigma_{\vec{K}-\vec{q}} \right)^2 D(\vec{q}, t_1 - t_2) \right. \right. \\
&+ \left. \left(v_{\vec{q}} \sigma_{\vec{K}} \alpha_{\vec{K}-\vec{q}} - u_{\vec{q}} \alpha_{\vec{K}} \sigma_{\vec{K}-\vec{q}} \right)^2 D(-\vec{q}, t_2 - t_1) \right] G_0^- \left(\vec{K} - \vec{q}, t_1 - t_2 \right) \\
&+ \left[\left(u_{\vec{q}} \sigma_{\vec{K}} \beta_{\vec{K}-\vec{q}} + v_{\vec{q}} \gamma_{\vec{K}-\vec{q}} \alpha_{\vec{K}} \right)^2 D(\vec{q}, t_1 - t_2) \right. \\
&+ \left. \left. \left(v_{\vec{q}} \sigma_{\vec{K}} \beta_{\vec{K}-\vec{q}} + u_{\vec{q}} \gamma_{\vec{K}-\vec{q}} \alpha_{\vec{K}} \right)^2 D(-\vec{q}, t_2 - t_1) \right] G_0^+ \left(\vec{K} - \vec{q}, t_1 - t_2 \right) \right\} \quad (\text{A11})
\end{aligned}$$

By going to the frequency representation with a Fourier transformation with respect to the time, as in Eq. (23), the following expression is retrieved for the second order correction to the Green's function of the lower quasiparticle branch:

$$\begin{aligned}
G^{-(2)}(\vec{k}, \omega) &= \frac{i}{\hbar^2} \int_{-\infty}^{\infty} \frac{d\nu}{2\pi} \sum_{\vec{q}} G_0^- \left(\vec{K}, \omega \right) G_0^- \left(\vec{K}, \omega \right) \\
&\times \left\{ \left[\left(u_{\vec{q}} \sigma_{\vec{K}} \alpha_{\vec{K}-\vec{q}} - v_{\vec{q}} \alpha_{\vec{K}} \sigma_{\vec{K}-\vec{q}} \right)^2 D(\vec{q}, \nu) \right. \right. \\
&+ \left. \left(v_{\vec{q}} \sigma_{\vec{K}} \alpha_{\vec{K}-\vec{q}} - u_{\vec{q}} \alpha_{\vec{K}} \sigma_{\vec{K}-\vec{q}} \right)^2 D(-\vec{q}, -\nu) \right] G_0^- \left(\vec{K} - \vec{q}, \omega - \nu \right) \\
&+ \left[\left(u_{\vec{q}} \sigma_{\vec{K}} \beta_{\vec{K}-\vec{q}} + v_{\vec{q}} \gamma_{\vec{K}-\vec{q}} \alpha_{\vec{K}} \right)^2 D(\vec{q}, \nu) \right. \\
&+ \left. \left. \left(v_{\vec{q}} \sigma_{\vec{K}} \beta_{\vec{K}-\vec{q}} + u_{\vec{q}} \gamma_{\vec{K}-\vec{q}} \alpha_{\vec{K}} \right)^2 D(-\vec{q}, -\nu) \right] G_0^+ \left(\vec{K} - \vec{q}, \omega - \nu \right) \right\} \quad (\text{A12})
\end{aligned}$$

This can be written as a function of the self energy $\Sigma^{-(2)}(\vec{k}, \omega)$:

$$G^{-(2)}(\vec{k}, \omega) = G_0^- \left(\vec{K}, \omega \right) \Sigma^{-(2)}(\vec{k}, \omega) G_0^- \left(\vec{K}, \omega \right), \quad (\text{A13})$$

showing that the self energy is given by:

$$\begin{aligned}
\Sigma^{-(2)}(\vec{k}, \omega) = & \frac{i}{\hbar^2} \int_{-\infty}^{\infty} \frac{d\nu}{2\pi} \sum_{\vec{q}} \left\{ \left[\left(u_{\vec{q}} \sigma_{\vec{K}} \alpha_{\vec{K}-\vec{q}} - v_{\vec{q}} \alpha_{\vec{K}} \sigma_{\vec{K}-\vec{q}} \right)^2 D(\vec{q}, \nu) \right. \right. \\
& + \left. \left(v_{\vec{q}} \sigma_{\vec{K}} \alpha_{\vec{K}-\vec{q}} - u_{\vec{q}} \alpha_{\vec{K}} \sigma_{\vec{K}-\vec{q}} \right)^2 D(-\vec{q}, -\nu) \right] G_0^-(\vec{K} - \vec{q}, \omega - \nu) \\
& + \left[\left(u_{\vec{q}} \sigma_{\vec{K}} \beta_{\vec{K}-\vec{q}} + v_{\vec{q}} \gamma_{\vec{K}-\vec{q}} \alpha_{\vec{K}} \right)^2 D(\vec{q}, \nu) \right. \\
& + \left. \left. \left(v_{\vec{q}} \sigma_{\vec{K}} \beta_{\vec{K}-\vec{q}} + u_{\vec{q}} \gamma_{\vec{K}-\vec{q}} \alpha_{\vec{K}} \right)^2 D(-\vec{q}, -\nu) \right] G_0^+(\vec{K} - \vec{q}, \omega - \nu) \right\}. \quad (\text{A14})
\end{aligned}$$

Using the Dyson equation it is clear that the self energy $\Sigma(\vec{k}, \omega)$ from (24) can be approximated by the self energy (A14). The frequency integral in Eq. (A14) can be done with contour integration which finally results in the expression (25a) for the self energy of the lower branch. For the upper branch a similar calculation can be done which results in expression (25b).

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